# Fatigue & Fracture Worked Example 2 – Fracture

Department of Mechanical, Materials & Manufacturing Engineering MMME2053 – Mechanics of Solids



# Worked Example 2

## **Thumbnail Crack in Plate**

A large high carbon steel plate with a thumbnail crack, as shown below, for which  $K = 1.2\sigma\sqrt{\pi a}$ , has a fracture toughness of 150 MPa $\sqrt{m}$  and yield stress,  $\sigma_{\gamma}$ , of 230 MPa.



#### Problem

If plate is loaded in tension with an applied stress,  $\sigma = \frac{2}{3}\sigma_y$ , determine the critical initial crack size assuming linear elastic material.

### Solution

Expression for Stress Intensity Factor for this geometry is:

$$K = 1.2\sigma\sqrt{\pi a}$$

Assuming the crack length, a, is at the critical length for causing fracture of the sample,  $a_{cr}$ , this expression can be rewritten as:

$$K_{\rm cr} = 1.2\sigma\sqrt{\pi a_{\rm cr}}$$

Substituting  $\sigma = \frac{2}{3}\sigma_y$  into this:

$$K_{\rm cr} = \frac{4}{5}\sigma_y \sqrt{\pi a_{\rm cr}}$$

Re-arranging for  $a_{cr}$  and substituting in values for  $K_{cr}$  and  $\sigma_y$ :

$$a_{\rm cr} = \frac{1}{\pi} \left( \frac{K_{\rm cr}}{\frac{4}{5}\sigma_y} \right)^2 = \frac{1}{\pi} \left( \frac{150}{\frac{4}{5} \times 230} \right)^2 = 0.305 \text{ m} = 305 \text{ mm}$$